

Nucleon Form Factors and Hidden Symmetry in Holographic QCD

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Abstract

The vector dominance of the electromagnetic form factors both for mesons and baryons arises naturally in holographic QCD, where both the number of colors and the 't Hooft coupling are taken to be very large, offering a bona-fide derivation of the notion of vector dominance. The crucial ingredient for this is the infinite tower of vector mesons in the approximations made which share features that are characteristic of the quenched approximation in lattice QCD. We approximate the infinite sum by contributions from the lowest four vector mesons of the tower which turn out to saturate the charge and magnetic moment sum rules within a few percent and compute them totally free of unknown parameters for momentum transfers $Q^2 \lesssim 1 \text{ GeV}^2$. We identify certain observables that can be reliably computed within the approximations and others that are not, and discuss how the improvement of the latter can enable one to bring holographic QCD closer to QCD proper.

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I. INTRODUCTION

Recently, there has been active study on the gravity dual models of quantum chromodynamics (QCD), named as holographic QCD (hQCD). One of the consequences of hQCD is the vector meson dominance in the low energy dynamics of QCD, where not just the ground-state vector mesons $V^{gs} = \rho, \omega, \phi$ contribute, as originally proposed by Sakurai in the 1960s [1], but the whole tower of vector mesons (denoted V^∞), including all excited states, do contribute. If this new vector meson dominance is verified experimentally, it will indicate strongly that QCD has a hidden symmetry, which is best described by Yang-Mills gauge fields in a five-dimensional spacetime with a warped factor dependent on the extra fifth dimension, and which, reduced to 4 dimensions, manifest themselves as an infinite tower of vector and axial vector mesons. How the 5D spacetime is curved at the hadronic scale can be inferred directly from the spectra of vector mesons and their role in the response functions of hadrons to external fields.

The vector meson dominance with the ground-state mesons (V^{gs}) has been a powerful tool in studying electroweak structure of hadrons in nuclear and hadron physics. It has met with a remarkable phenomenological success, ranging from electromagnetic form factors of light-quark hadrons to deep-inelastic scattering in the diffraction region of small $x = Q^2/W^2 \ll 1$. (See [2] for a recent discussion on this.) On the theoretical side, however, the situation has been less than satisfactory. First of all there is no bona-fide “derivation” of VMD^{gs} with the ground-state vector mesons from first principles. The formulation of VMD^{gs} employing an operator identity known as current-field identity [3, 4] – which was anchored on gauge principle – gave a natural explanation of “universality” of electromagnetic coupling $g_{\rho\pi\pi} = g_{\rho\rho\rho} = g_{\rho NN}$. However there was a glaring defect in describing nucleon electromagnetic (EM) form factors in terms of VMD^{gs} , which destroyed the notion of universality. This shortcoming has become more prominent with the advent of precision measurements of nucleon form factors at the JLab.

The problem, briefly stated, is as follows. While VMD^{gs} works remarkably well to large momentum transfers for the pion form factor in both the space-like and time-like regimes, it fails badly for the nucleon. Although the early nucleon form factor measurements were interpreted as empirical evidence for an isoscalar vector meson, $\omega \rightarrow 3\pi$, by Nambu [5] in 1957, and for an isovector meson, $\rho^0 \rightarrow 2\pi$, by Frazer and Fulco [6] in 1959, it has

subsequently been shown that nucleon EM form factors cannot be described satisfactorily by the exchange of V^{gs} alone but requires an important additional component representing an “intrinsic core” of size $\sim 0.3 - 0.4$ fm [7, 8]. A two-component picture implementing the latter to the standard VMD^{gs} has been modeled by including a direct photon coupling to the nucleon that has an intrinsic core of an unspecified source [9] or a bag of confined chiral quarks [10]. The consequence of this two-component picture was that the universality encoded in VMD^{gs} is lost, i.e., $g_{\rho\pi\pi} \neq g_{\rho NN}$ and hence vector dominance is violated.

It was shown in [11] that in the holographic, instanton picture of the nucleon, which is effectively the Skyrmion picture drastically modified by the inclusion of infinite number of vector mesons and axial-vector mesons, a full vector dominance re-emerges. This restores the universality relation – lost with V^{gs} alone – in a form that involves the whole tower. In this paper, we report how the model for the nucleon EM form factors fares with nature at low momentum transfers. What we are doing here can be considered as a *prediction* – and not a postdiction – of the hQCD model since the calculation involves *no free parameters*: all the pertinent parameters of the action in the approximations adopted, i.e., large N_c and $\lambda = g_s^2 N_c$ limits, are completely fixed in the meson sector [12]. This model was found [11] to give a satisfactory description of chiral dynamics that can be reliably treated – such as the axial coupling constant, isovector magnetic moment etc. – in the quenched approximation of QCD.

II. VECTOR DOMINANCE FOR NUCLEON FORM FACTORS

The nucleon form factors are defined from the matrix elements of the external currents,

$$\langle p' | J^\mu(x) | p \rangle = e^{iqx} \bar{u}(p') \mathcal{O}^\mu(p, p') u(p), \quad (1)$$

where $q = p' - p$ and $u(p)$ is the nucleon spinor of momentum p . By the Lorentz invariance and the current conservation we expand the operator \mathcal{O}^μ , assuming the CP invariance, as

$$\mathcal{O}^\mu(p, p') = \gamma^\mu \left[\frac{1}{2} F_1^s(Q^2) + F_1^a(Q^2) \tau^a \right] + i \frac{\sigma^{\mu\nu} q_\nu}{2m_N} \left[\frac{1}{2} F_2^s(Q^2) + F_2^a(Q^2) \tau^a \right], \quad (2)$$

where $m_N \simeq 940$ MeV is the nucleon mass and $\tau^a = \sigma^a/2$. F_1^s and F_2^s are the Dirac and Pauli form factors for iso-scalar current respectively, and F_1^a , F_2^a are for iso-vector currents.

Being matrix elements, the form factors contain all one-particle irreducible diagrams for two nucleons and one external current and thus very difficult to calculate from QCD. How-

ever, the anti-de Sitter/conformal field theory (AdS/CFT) correspondence, or gravity/gauge theory correspondence, found in certain types of string theory, enables us to compute such non-perturbative quantities like hadron form factors with mild approximations [13, 14].

According to this correspondence, the low energy effective action of the gravity dual of QCD becomes the generating functional for the correlators of an operator \mathcal{O} in QCD in the large N_c limit:

$$e^{iS_{5D}^{\text{eff}}[\phi(\epsilon, x)]} = \left\langle \exp \left[i \int_x \phi_0 \mathcal{O} \right] \right\rangle_{\text{QCD}}, \quad (3)$$

where $\phi(z, x)$ is a bulk field, acting as a source for \mathcal{O} when evaluated at the UV boundary $z = \epsilon$. Furthermore the normalizable modes of the bulk field are identified as the physical states in QCD, created by the operator \mathcal{O} .

A gravity dual of low energy QCD with massless flavors is proposed in the quenched approximation by Sakai and Sugimoto (SS) [12]. Later, the holographic dual model of spin- $\frac{1}{2}$ baryons, or nucleons, in the SS model with two flavors is constructed [11, 15] by introducing a bulk baryon field, whose effective action is given in the conformal coordinate (x, w) as

$$S_{5D}^{\text{eff}} = \int_{x, w} \left[-i\bar{\mathcal{B}}\gamma^m D_m \mathcal{B} - im_b(w)\bar{\mathcal{B}}\mathcal{B} + \kappa(w)\bar{\mathcal{B}}\gamma^{mn}F_{mn}^{SU(2)_I}\mathcal{B} + \dots \right] + S_{\text{meson}}, \quad (4)$$

where \mathcal{B} is the 5D bulk baryon field, D_m is the gauge covariant derivative and S_{meson} is the effective action for the mesons [16]. Using the instanton nature of baryon, the coefficients $m_b(w)$ and $\kappa(0)$ can be reliably calculated in string Theory [11, 15]. Especially, the coefficient of the magnetic coupling is estimated to be

$$\kappa(w) \simeq \frac{0.18N_c}{M_{KK}}, \quad (5)$$

where $M_{KK} \simeq 0.94$ GeV is the ultraviolet cutoff of the SS model. The ellipsis in the effective action (4) denotes higher derivative operators, whose coefficients are difficult to estimate, but are suppressed at low energy, $E < M_{KK}$. The important point is that the magnetic coupling involves only the non-Abelian part of flavor symmetry $SU(2)_I$, with Abelian $U(1)_B$ being absent, due to the non-Abelian nature of instanton-baryons.

Though the exact correspondence between the gravity dual and QCD is not established yet, we compute the electromagnetic (EM) form factors for the nucleons in the SS model by assuming the correspondence. We first need to identify the dual bulk field of the external EM current, which is nothing but the bulk photon field. Since the electric charge operator

is the sum of isospin and the baryon operator,

$$Q_{\text{em}} = I_3 + \frac{1}{2}B, \quad (6)$$

we have to identify a combination of A_μ^3 and A_μ^B , the third component of the isospin gauge field and the $U(1)_B$ gauge field, respectively, as the photon field. Then all baryon bilinear operators in the effective action that couple to either $U(1)_B$ gauge fields or $SU(2)_I$ gauge fields will contribute to the EM form factors.

We now write the (nonnormalizable) photon field as

$$A_\mu(x, w) = \int_q A_\mu(q) A(q, w) e^{iqx}, \quad (7)$$

with boundary conditions that $A(q, w) = 1$ and $\partial_w A(q, w) = 0$ at the UV boundary, $w = \pm w_{\text{max}}$ and the (normalizable) bulk baryon field as

$$\mathcal{B}(w, x) = \int_p [f_L(w) u_L(p) + f_R(w) u_R(p)] e^{ipx}. \quad (8)$$

These 5D wave functions, $A(q, w)$ and $f_{L,R}(w)$, are determined by solving the equation of motion from our action (4). Then, using the correspondence (3), one can read off the form factors. We find for nucleons the Dirac form factor $F_1(Q^2) = F_{1\text{min}} Q_{\text{em}} + F_{1\text{mag}} I_3$ with ($Q^2 \equiv -q^2$)

$$F_{1\text{min}}(Q^2) = \int_{-w_{\text{max}}}^{w_{\text{max}}} dw |f_L(w)|^2 A(q, w), \quad (9)$$

$$F_{1\text{mag}}(Q^2) = 2 \int_{-w_{\text{max}}}^{w_{\text{max}}} dw \kappa(w) |f_L(w)|^2 \partial_w A(q, w), \quad (10)$$

where $F_{1\text{min}}$ is from the minimal coupling, and $F_{1\text{mag}}$ the magnetic coupling. Similarly the Pauli form factor is given as $F_2(Q^2) = F_2^3(Q^2) I_3$ with

$$F_2^3(Q^2) = 4 m_N \int_{-w_{\text{max}}}^{w_{\text{max}}} dw \kappa(w) f_L^*(w) f_R(w) A(q, w), \quad (11)$$

which comes solely from the magnetic coupling. We note that the form factors (9), (10) and (11) have corrections coming from the higher order operators in the effective action (4), which are suppressed by powers of E/M_{KK} at low energy. However we emphasize that our result contains full quantum effects in the large N_c limit, because it is computed from the generating functional for one-particle irreducible correlation functions, summed over all loops, prescribed by the AdS/CFT correspondence.

Alternatively, we can replace the form factors by an infinite sum of vector-meson exchanges [13, 17], if we expand the nonnormalizable photon field in terms of the normalizable vector meson ψ_{2k+1} of mass m_{2k+1} as

$$A(q, w) = \sum_k \frac{g_{v^{(k)}} \psi_{(2k+1)}(w)}{Q^2 + m_{2k+1}^2}, \quad (12)$$

where the decay constant of the k -th vector mesons is given as $g_{v^{(k)}} = m_{2k+1}^2 \zeta_k$ with

$$\zeta_k = \frac{\lambda N_c}{108\pi^3} M_{KK} \int_{-w_{max}}^{w_{max}} dw \frac{U(w)}{U_{KK}} \psi_{(2k+1)}(w), \quad (13)$$

where U_{KK} is a parameter of the SS model and

$$dw = \frac{3}{2} \frac{U_{KK}^{1/2}}{M_{KK}} \frac{dU}{\sqrt{U^3 - U_{KK}^3}}. \quad (14)$$

The resulting EM form factors then take the form

$$F_1(Q^2) = F_{1min} Q_{em} + F_{1mag} \tau^3 = \sum_{k=1}^{\infty} \left(g_{V,min}^{(k)} Q_{em} + g_{V,mag}^{(k)} \tau^3 \right) \frac{\zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2}, \quad (15)$$

$$F_2(Q^2) = F_2^3(Q^2) \tau^3 = \tau^3 \sum_{k=1}^{\infty} \frac{g_2^{(k)} \zeta_k m_{2k+1}^2}{Q^2 + m_{2k+1}^2}, \quad (16)$$

where

$$g_{V,min}^{(k)} = \int_{-w_{max}}^{w_{max}} dw |f_L(w)|^2 \psi_{(2k+1)}(w) \quad (17)$$

$$g_{V,mag}^{(k)} = 2 \int_{-w_{max}}^{w_{max}} dw \kappa(w) |f_L(w)|^2 \partial_w \psi_{(2k+1)}(w),$$

$$g_2^{(k)} = 4m_N \int_{-w_{max}}^{w_{max}} dw \kappa(w) f_L^*(w) f_R(w) \psi_{(2k+1)}(w). \quad (18)$$

The authors of [11] noted that the sum rules for the electric charge and the magnetic moment given in the large N_c and λ limit are well saturated by first four vector mesons. After shifting $N_C \rightarrow N_C + 2$ for the magnetic coupling as described in [11] [18], one finds that the sum rules are saturated for protons (and similarly for neutrons) within a few % when we take $N_C = 3$:

$$F_1^p(0) \equiv 1 \simeq \sum_{k=1}^4 \left(g_{V,min}^{(k)} + \frac{1}{2} \cdot \frac{N_C + 2}{N_C} \cdot g_{V,mag}^{(k)} \right) \zeta_k = 1.04,$$

$$F_2^p(0) \equiv \mu_p - 1 \simeq \frac{1}{2} \cdot \frac{N_C + 2}{N_C} \cdot \sum_{k=1}^4 g_2^{(k)} \zeta_k = 1.66. \quad (19)$$

Recovering the mass unit M_{KK} , we obtain, taking $N_c = 3$ and $f_\pi \simeq 93$ MeV as determined in the meson sector, from Table 2 of [11]

$$F_1^p(Q^2) \simeq \frac{0.958 M_{KK}^2}{Q^2 + 0.67 M_{KK}^2} - \frac{1.230 M_{KK}^2}{Q^2 + 2.87 M_{KK}^2} - \frac{0.628 M_{KK}^2}{Q^2 + 6.59 M_{KK}^2} + \frac{1.585 M_{KK}^2}{Q^2 + 11.8 M_{KK}^2} \quad (20)$$

$$F_{1\text{mag}}(Q^2) \simeq -\frac{0.248 M_{KK}^2}{Q^2 + 0.67 M_{KK}^2} + \frac{2.602 M_{KK}^2}{Q^2 + 2.87 M_{KK}^2} - \frac{5.777 M_{KK}^2}{Q^2 + 6.59 M_{KK}^2} + \frac{5.153 M_{KK}^2}{Q^2 + 11.8 M_{KK}^2} \quad (21)$$

$$F_2^p(Q^2) \simeq \frac{1.855 M_{KK}^2}{Q^2 + 0.67 M_{KK}^2} - \frac{4.587 M_{KK}^2}{Q^2 + 2.87 M_{KK}^2} + \frac{4.547 M_{KK}^2}{Q^2 + 6.59 M_{KK}^2} - \frac{2.390 M_{KK}^2}{Q^2 + 11.8 M_{KK}^2}. \quad (22)$$

Equations (20), (21) and (22) constitute the main ingredients for the analysis that follows.

The observable quantities we are interested in are the Sachs form factors for protons and neutrons defined by

$$G_M^p(Q^2) = F_1^p(Q^2) + F_2^p(Q^2), \quad (23)$$

$$G_E^p(Q^2) = F_1^p(Q^2) - \frac{Q^2}{4m_N^2} F_2^p(Q^2), \quad (24)$$

$$G_M^n(Q^2) = -\frac{1}{2} F_{1\text{mag}}(Q^2) - F_2^p(Q^2), \quad (25)$$

$$G_E^n(Q^2) = -\frac{1}{2} F_{1\text{mag}}(Q^2) + \frac{Q^2}{4m_N^2} F_2^p(Q^2), \quad (26)$$

where we used the fact that the Pauli form factor of neutron is $F_2^n(Q^2) = -F_2^p(Q^2)$ in holographic QCD, found by the authors [11, 15].

III. NUMERICAL RESULTS

Before we make the estimates of various physical quantities, let us briefly review the standard practice in comparing theoretical (model) calculations with experiments.

All the parameters in our approach are fixed in the meson sector, so there are no free parameters to adjust. In phenomenological models based on the vector dominance by the ground state V^{gs} [9] or on the skyrmion core surrounded by the V^{gs} cloud [19], however, one takes into account several features that are extraneous to the models. For instance, perturbative QCD tells us that at asymptotic momentum transfer, $F_i(Q^2)$ should scale as $F_1 \sim Q^{-4}$ and $F_2 \sim Q^{-6}$. Our form factors (20) and (22) do not possess these scaling features. Furthermore, even if one includes one or more excited vector mesons in the model, one needs to account at least for the fact that the ground-state ρ has a large width. All these are included by hand in phenomenological model fits. If we wished to fit the experimental

data to $Q^2 \sim 10 \text{ GeV}^2$ available in the literature [20], these properties should also be taken into account. The hQCD we are studying cannot handle any of these at present since they require calculating $1/N_c$ and $1/\lambda$ corrections and include short-distance interactions given by perturbative QCD. Since our aim here is to see how the theory in the given approximations fares with no unknown parameters, we will eschew introducing arbitrary phenomenological factors. We will therefore limit our kinematics to $Q^2 \lesssim M_{KK}^2 \sim 1 \text{ GeV}^2$.

A. Charge and Magnetic Radii

Since we have seen that static quantities involving $F_i(0)$ come out in good agreement with experiments [11], the next low-momentum quantity we can calculate are the electric charge radius r_e and magnetic radius r_m . We will focus on the proton structure. Taking $M_{KK} \approx m_N \simeq 0.94 \text{ MeV}$ as fixed in the meson sector [12], we readily obtain

$$r_{pe}^2 \equiv -6 \frac{d}{dQ^2} [\ln G_E^p(Q^2)] \Big|_{Q^2=0} \simeq (0.796 \text{ fm})^2, \quad (27)$$

$$r_{pm}^2 \equiv -6 \frac{d}{dQ^2} [\ln G_M^p(Q^2)] \Big|_{Q^2=0} \simeq (0.744 \text{ fm})^2. \quad (28)$$

These are to be compared with the experimental values [20]

$$\tilde{r}_{pe}^{exp} = 0.895 \pm 0.018 \text{ fm}, \quad \tilde{r}_{pm}^{exp} = 0.855 \pm 0.035 \text{ fm}. \quad (29)$$

We first note that both of the predicted radii are $\sim 0.1 \text{ fm}$ smaller than the experimental values. This deviation is expected since our estimate is reliable only when the number of colors and also the 't Hooft coupling λ are very large. In the SS model of QCD, the “core” radius of the nucleons scales as $1/\sqrt{\lambda}$ and therefore the resulting core size $\Delta r \sim 0.1 \text{ fm}$ must be due to the subleading corrections, which are extremely difficult to estimate. What is noteworthy is that the core size which comes out to be $\sim 0.4 \text{ fm}$ when only the ground-state vector mesons V^{gs} are taken into account [9], shrinks to $\sim 0.1 \text{ fm}$ in the presence of the tower of vector mesons, here encapsulated in the three higher-lying members. The full account of the tower may shrink the “core” size further with the higher tower playing the role of a major part of the intrinsic core or quark-bag degrees of freedom.

We should note however that the underestimate of the root-mean-square radii while static quantities, e.g., magnetic moments, g_A etc. come out close to experimental values [15] is

a generic feature of quenched approximations as noticed in quenched lattice calculations of vector and axial nucleon form factors [21].

B. $\mu_p G_E^p(Q^2)/G_M^p(Q^2)$

We expect the ratios of form factors to be less sensitive than the form factors themselves to $1/N_c$ and $1/\lambda$ corrections and also to additional form factors representing asymptotic scaling which manifest themselves in the “core size.” We therefore look at the ratio

$$R_1(Q^2) = \mu_p G_E^p(Q^2)/G_M^p(Q^2). \quad (30)$$

The result is plotted in Fig.1. Given that our calculations are valid in the large N_c and λ

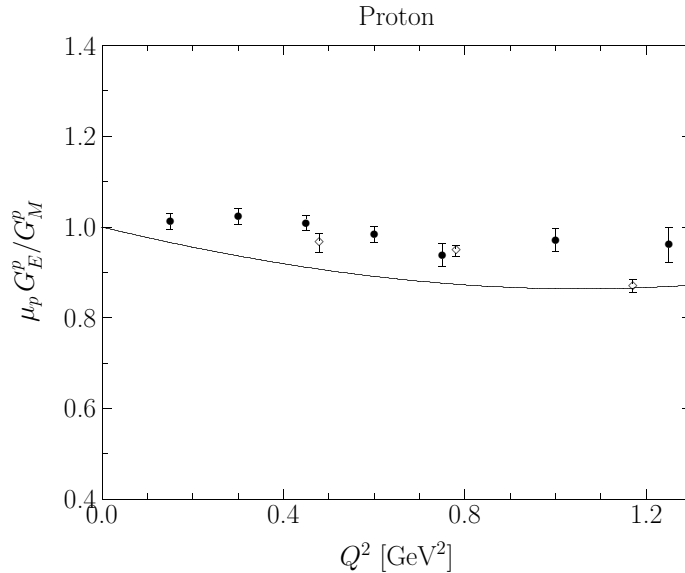


FIG. 1: The open circles are the polarization measurements at JLab [23] and the filled circles are the data taken from [24]. The solid line is the prediction in the SS model.

limits and also in the chiral limit, and above all that there are no free parameters [22], the agreement with experiments within, say, $\lesssim 10\%$ is quite surprising. However, the fact that the corrections cancel out largely in the ratio suggests that the R_1 belongs to the class of observables for which quenched approximations are applicable. The $\lesssim 10\%$ underestimate can be readily understood as explained below.

C. $G_M^p(Q^2)/\mu_p G_D(Q^2)$ and $G_E^p(Q^2)/G_D(Q^2)$

In the past before the advent of precision data, more recently from the JLab, the standard parametrization of the nucleon form factors was done in terms of a dipole form. The dipole form clearly showed that the nucleon form factors could not be understood in terms of the monopole form alone given by the vector dominance with V^{gs} . New data, particularly those from polarization-transfer [20], show that the dipole form factor starts deviating for $Q^2 \gtrsim 2$ GeV², so there is nothing sacred with the dipole form. However for $Q^2 \lesssim 1$ GeV², it is still a good standard measure. We therefore examine the ratios

$$R_2^m(Q^2) = G_M^p(Q^2)/\mu_p G_D(Q^2), \quad (31)$$

$$R_2^e(Q^2) = G_E^p(Q^2)/G_D(Q^2) \quad (32)$$

with the dipole form factor parameterized as $G_D = 1/(1 + Q^2/0.71)^2$. These are plotted in Fig. 2. Within the range of momentum transfers considered, the theoretical G_M/μ_p is seen

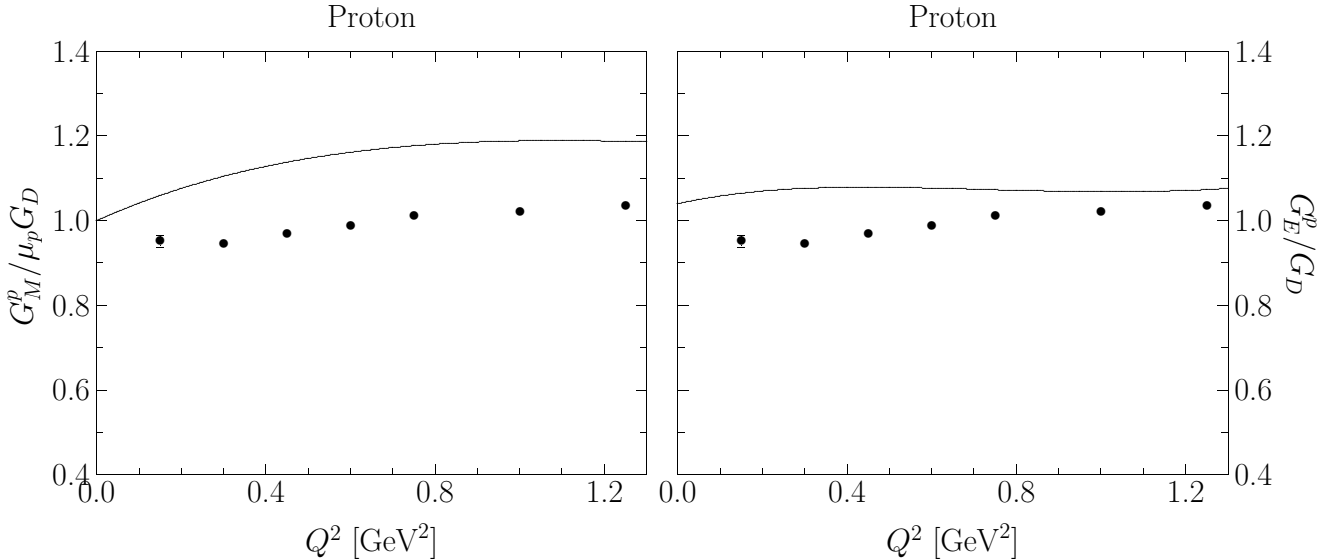


FIG. 2: The predictions are given by the solid lines and the filled circles are the data taken from [24]. The left panel corresponds to the magnetic form factor of proton and the right panel corresponds to the electric form factor of proton.

to overshoot the experimental by about $\sim 14\%$ and the corresponding G_E by $\lesssim 10\%$. These results can be simply understood by the differences between the predicted radii ~ 0.74 fm and ~ 0.80 fm for G_M and G_E , respectively and the radius given by the dipole form factor

~ 0.81 fm. The conclusion then is that the mechanism that accounts for the defects in size $\Delta r_M \sim 0.07$ fm and $\Delta r_E \sim 0.01$ fm are responsible for the small observed discrepancies.

D. $Q^2 F_2(Q^2)/F_1(Q^2)$

To identify the source of the small deviation observed above, it is instructive to look at the ratio of the Pauli form factor over the Dirac form factor:

$$R_3(Q^2) = Q^2 F_2(Q^2)/F_1(Q^2) \quad (33)$$

with Q^2 given in units of GeV^2 . This ratio is plotted in Fig. 3. We observe that the predicted

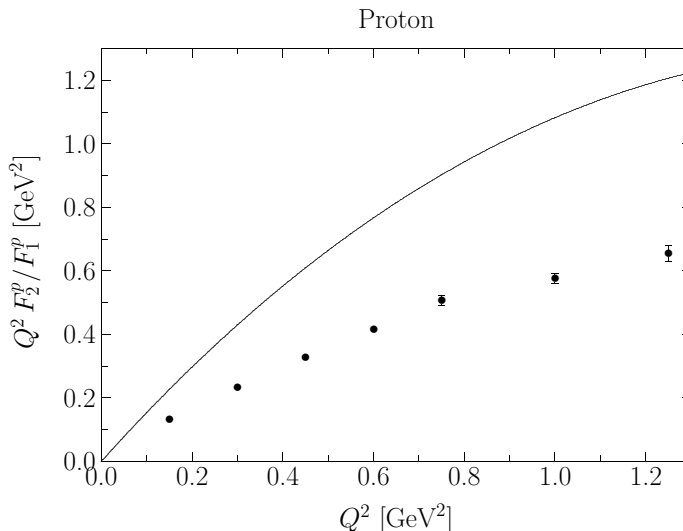


FIG. 3: The filled circles are the data taken from [24]. The solid line is the prediction in the SS model with $M_{KK} = 0.94$ GeV.

Pauli form factor drops too slowly compared with the Dirac form factor at large Q^2 . As noted above, QCD proper demands that asymptotically $F_2/F_1 \sim Q^{-2}$ but this feature is missing in hQCD in the large N_c and λ limit. We believe that this feature gives a simple explanation as to how the small discrepancies arise in Fig. 2: A reduction of F_2 by the amount indicated in Fig. 3 would bring the theoretical curves closer to the dipole form factor. Furthermore the overestimate of F_2 for non-zero Q^2 can account for the $\lesssim 10\%$ undershooting of the ratio R_1 as one can see from how F_2 enters into G_M^p [Eq. (23)] and G_E^p [Eq. (24)].

IV. VIOLATION OF VMD^{gs}

That hQCD à la SS restores vector dominance to the nucleon structure on the same footing as for the pion raises an interesting question on the possible role of the higher members of the tower of vector mesons and their relation to the predicted violation [25, 26] of VMD^{gs} in hot and dense matter. This is a current topical issue in holographic QCD [27] prompted by experimental developments on heavy-ion collisions at CERN and RHIC where hadronic matter is heated to high temperature and compressed to high density.

To bring out the basic issue, it is illuminating to consider the hidden local symmetry (HLS) theory of Harada and Yamawaki [28] which in some sense can be interpreted [29] to be a truncated version of the SS model in which all higher members of the tower than the ground state vector mesons (V^{gs}) are integrated out and the high-energy sector is matched at a scale $\sim M_{KK}$ to QCD via various current-current correlators. One can also view the V^{gs} that figure in HLS theory as “emergent” fields [30] which when extended to an infinite number, leads to a dimensionally deconstructed 5D Yang-Mills theory of QCD [31]. The emergence of local (nonabelian) gauge invariance is analogous to the emergence of a $U(1)$ gauge degree of freedom in the CP^{n-1} model. Here the gauge degrees of freedom are lodged in the chiral field $U = e^{2i\pi/F_\pi} = \xi_L^\dagger \xi_R$ with $\xi_{L,R} = e^{\mp i\pi/F_\pi} e^{i\sigma/F_\sigma}$ where π is the pion field and σ is the scalar multiplet that makes up the redundant degree of freedom. This redundancy is elevated to a gauge degree of freedom with the vectors V^{gs} emerging as a (hidden) gauge field. What is important for our discussion here is that this HLS theory has a fixed point to which the system is driven constrained by chiral symmetry of QCD. At that fixed point (called “vector manifestation (VM) fixed point”), the parameters of the HLS Lagrangian take the values

$$(g^*, f_\pi^*, a^*) = (0, 0, 1) \quad (34)$$

where g is the hidden gauge coupling, f_π is the physical pion decay constant and a is the ratio of decay constants

$$a = \left(\frac{F_\pi}{F_\sigma} \right)^2. \quad (35)$$

The Harada-Yamawaki theory with the VM fixed point (34) is called “HLS/VM.”

In this theory, the isovector photon coupling is given by

$$\delta\mathcal{L} = e\mathcal{A}_{EM}^\mu \left(-2aF_\pi^2 \text{Tr}[g\rho_\mu \hat{Q}] + 2i(1 - a/2) \text{Tr}[J_\mu \hat{Q}] \right), \quad (36)$$

where \hat{Q} is the quark charge matrix, ρ_μ is the lowest-lying iso-vector vector meson and J_μ is the iso-vector vector current made up of the chiral field ξ . The first term of (36) represents the photon coupling through a ρ and the second term the direct coupling. The vector dominance in this theory (VMD^{gs}) is obtained when $a = 2$ for which the well-known KSRF relation for the ρ meson holds, i.e., $m_\rho^2 = af_\pi^2 g^2 = 2f_\pi^2 g^2$.

The crucial observation in this theory [32] is that $a = 2$ that leads to VMD^{gs} is *not* on the RG trajectory connected to the fixed point. In fact, $a = 2$ is found to lie on an unstable trajectory and any infinitesimal perturbation moves the system away from the value $a = 2$. In nature, this is the case when hadronic matter is heated or compressed [25], with the chiral phase transition that occurs when the quark condensate goes to zero coinciding with the $a = 1$ point. At this point, VMD^{gs} is maximally violated.

We can now interpret what we found in the previous section in hQCD in terms of HLS/VM as follows based on two observations: (a) In matter-free space, the pion form factor is given by $a = 2$, so it is vector-dominated in the Sakurai sense. What is somewhat surprising is that the VMD[∞] in hQCD (nearly completely saturated by the four lowest members of the tower) also describes the pion form factor well. Here the two pictures seem to give quite similar results for the pion form factors. However this may be coincidental, for the RG analysis has shown that when the system is heated and/or compressed, a departs quickly from 2 and moves to the fixed point $a = 1$ [28] thereby violating VMD^{gs}; (b) as for the nucleon, the VMD^{gs} is maximally violated already in matter-free vacuum: the two-component models imply that $a \approx 1$, with a phenomenologically favored 50-50 coupling to the vector meson and the “core” represented by the second term of (36) as discussed in [10].

The above two observations are suggestive of that the higher members of the tower in hQCD could be playing an important role in hot/dense medium. The key observation in HLS/VM is the decrease of the quark condensate $\langle \bar{q}q \rangle$ as well as of the vector meson mass $m_{V^{gs}}$ as temperature/density increases. But this feature is missing in the current work on hQCD of SS in medium in which the large N_c and λ limit is taken and the current quark mass is left out. It would be interesting to see how the missing ingredient in the core size we find in the nucleon form factor is related to the chiral properties of hadrons seen in Harada-Yamawaki’s HLS/VM theory.

V. FURTHER COMMENTS

In this paper, we made a simple evaluation of the nucleon form factors that are vector-dominated by the infinite tower of vector mesons as derived from the instanton solution in the SS model [11]. With the infinite tower truncated to the four lowest vector mesons that saturate within a few % the zero-momentum sum rules, the (parameter-free) results, e.g., the proton radii, the proton form factors to the momentum transfer restricted by the KK mass $M_{KK} \sim 1$ GeV etc., come out to fare well with experiments. We could have done much better in comparing with the data by implementing ad-hoc phenomenological form factors that simulate the asymptotic freedom structure of QCD as has been done in the two-component models [9] and in the Skyrme model [19]. That would have allowed us to go beyond the kinematic regime $Q^2 \lesssim 1 \text{ GeV}^2$ and get a much better fit. But this was not the aim of our work. Our aim was to see whether the hQCD model as defined in given approximations and free of unknown parameters can resemble Nature. Our results answer this question in the positive and indicate how to improve the comparison with nature.

Assuming that the model *can* make meaningful predictions in the regime that QCD proper is unable to access, an interesting question to ask is what issues can be profitably addressed by the hQCD model. Indeed one of such issues is the role of the infinite tower in hot and dense medium discussed in the preceding section. Specifically it would be exciting if one could study how hadrons behave as temperature/density approaches the critical point where chiral phase transition is presumed to take place, currently a hot topic in the AdS/QCD circle [33]. In HLS/VM theory of Harada and Yamawaki, the strong violation of vector dominance with VMD^{gs} near the critical point is closely linked to the properties of the hadrons involved, e.g., vanishing vector meson (ρ , ω) masses and pion decay constant etc. Whether or not this description by HLS/VM theory is correct cannot at the moment be assessed by QCD: There are no QCD tools, including lattice, that allow one to access that regime. It seems plausible that in hQCD, it is the tower that will replace the role of a in HLS theory. Since in HLS/VM theory it is the quark condensate $\langle \bar{q}q \rangle$ that plays the key role, one would have to figure out how to correctly introduce the quark masses and quark condensates in hQCD to address the issue.

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- [18] This procedure does not involve mesonic loop graphs that bring in higher order corrections in the $1/N_c$ expansion. It naturally comes in a simple book-keeping of symmetry for the spin-isospin operators figuring in the Gamow-Teller operator and the isovector magnetic moment operator in electroweak processes as explained in [11].
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